Raising Scores in Problem Solving

Preparing Students to Meet the Standards

"RAISING SCORES" CONSISTS OF THREE PARTS:

* **Instruction and Assessment of Problem Solving**
* **Understanding the Difference in Skill, Concept and Problem Solving Tasks**
* **Profile of a Comprehensive Mathematics Program**

This document has been written to help you and your school develop a better understanding of mathematical problem solving and its place in mathematics within the classroom.

We believe that raising scores in problem solving will occur as a result of the integration of problem solving as it is described in this document.

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Introduction and Table of Contents

The purpose of this document is to help school communities examine their instruction and assessment of mathematical problem solving and to propose a system to help students meet and exceed the standards in problem solving.

I. Instruction and Assessment of Problem Solving

This graphic outlines the three stages in the problem solving process. It gives a visual image of what students do as they solve problems. The process is not a linear one and students move freely between stages.

These pages lay out a plan teachers could use in providing their students with direct instruction in problem solving. Bulleted lists summarize key actions by student, teacher and administrator to improve problem solving.

This section clarifies the differences between teaching problem solving and assessing the problem solving students can do on their own. It includes a comparison of the similarities and differences between the formal Vermont Portfolio Assessment Process and the New Standards Reference Exam problem solving tasks.

II. Understanding the Differences in Skill, Concept and Problem Solving Tasks

This section is designed to emphasize and clarify differences. Each of the grade level sections explores a single content area with examples of tasks designed to assess either skills, conceptual understanding or problem solving.

This section describes a comprehensive mathematics program designed to provide students with opportunities to meet the standard in problem solving. It focuses on the problem solving aspects of a comprehensive program.

Problem Solving Process

Problem Formulation
(Understand the Problem and Devise a Plan)

Read the Problem

Write an "I need to" statement [What is the question(s)]

Separate important and unimportant information
and identify any missing information

Think about the math concepts and problem solving strategies you know

Choose a strategy

Problem Implementation
(Carry Out the Plan)

Collect and explore data

Use your strategy and
solve the problem

Explain the approach you used,
document all your work
in a readable way and use
math language and representation

Make observations, connections,
and/or generalizations

Problem Conclusion
(Look Back at the Problem)

Summarize your conclusions and make any other observations,
connections or generalizations

Re-read the problem and check that your answer is correct and complete

Teaching Problem Solving

Problem solving should be a part of every student's daily work in math. Creating a problem-centered classroom will have a great impact on students' opportunity to learn and on their perception of mathematics.

The instructional format outlined below should help teachers develop a plan for teaching problem solving to their students. This is different from assessing the problem solving abilities their students have developed. While students are learning how to do problem solving they should have opportunities to see and discuss other strategies and to share connections.

Students should have a general framework for attacking any problem. Teaching the four step Polya problem solving method, which is outlined below, helps students become better problem solvers.

- **understand the problem**
- **devise a plan**
- **carry out the plan**
- **look back at the problem**

Problem formulation includes understanding the problem and devising a plan. Teaching problem formulation is a complex process. The teacher must be careful not to formulate the problem for students. Well-intentioned teachers who do this remove the formulation hurdle while trying to make sure that everyone understands what to do. Instead teachers need to give students practice in problem formulation, allowing for questions and discourse so that everyone makes an effort to understand the problem.

Understand the Problem

To successfully formulate a problem, students must first understand the problem they are being asked to solve. To find out what a problem means, students must understand how the words and phrases describe what is happening. Important information must be separated from the unimportant information, and any missing information must be identified. Students should understand the problem well enough to say it in their own words. Students must be able to state the question they have to answer to solve the problem. Asking students to write an "I need to" statement after careful reading is an essential first step.

Devise a Plan

Once students have established what they need to do, they should devise a plan to reach a solution. Different approaches to a problem are always possible, depending on the insights, skills and conceptual understanding of each student. Students need to see and be able to apply several basic problem solving strategies.

Some of the strategies the students will find helpful are these:

Identify a pattern

Guess and check

Write and use an equation

Work backwards

Make and use a diagram or model

Organize data in a table, chart, graph or plot

Solve a simpler related problem

Use objects or act out the problem

Eliminate possibilities using logical reasoning

These strategies should be taught directly to students throughout their K-12 mathematical experience. In addition students need the opportunity to solve problems using their own strategies and to learn from each other.

Carry Out the Plan

The next step in problem solving is to implement the strategies in the plan and solve the problem. The teacher can encourage students to use their chosen strategy by creating a classroom climate where all approaches are equally valued. It is important to have students document their work in a way that enables them to see at a glance what they've completed. To become independent problem solvers, students must learn to think and reason through solutions themselves. They will make more progress if the teacher answers their questions with questions, such as those in the attached list of generic questions (see appendix B).

Look Back at the Problem

The process of looking back at a problem does not only happen after the students have reached a solution. Observations, connections and generalizations can and will occur at any point in the problem solving process.

Students should learn to reread the problem and check the solution to see that it answers the question and meets the parameters given. Probing questions (see appendix B) might lead students to make an observation, discover a connection or generalize the problem to include other solutions.

Suggestions for Teachers

Teachers can help students become independent problem solvers by doing some or all of the following:

- Solve the problem and execute possible approaches to anticipate student difficulties (before assigning the task.)
- Think about the mathematical skills and conceptual understanding necessary to solve the problem.
- Encourage students to utilize the resources available to them. (calculators, math dictionaries, resource books, and manipulatives.)
- Regularly incorporate on-demand tasks, as well as longer tasks that students have an opportunity to think about for several days.
- Reduce anxiety by allowing communication and cooperation.
- Allow ample time for student discourse and reflection (both written and oral), so that students have an opportunity to learn from each other.

In addition, teachers can:

- Commit to fully implementing a standards-based program in their classroom that reflects the Vermont Framework of Standards and Learning Opportunities.
- Examine quizzes and tests; rewrite to provide better opportunities for students to conceptualize or problem solve.
- Balance direct instruction with constructivist learning to build competency in students.
- Take on different roles: guide, coach, observer, facilitator, model.
- Promote working and assessment portfolios for all students across all grades.
- Receive support from and provide support for colleagues.
- Establish exiting skills at various grade levels for mathematical language, representation, and problem solving strategies.
- Create task specific scoring guides to assess content.
- Recognize that small steps are valuable.
- Encourage students to accept challenges and persevere.
- Provide regular assessment and feedback.
- Embed problem solving into content instruction.

Worthwhile Tasks

Students need frequent, even daily, opportunities to solve problems. Worthwhile tasks need to be chosen that engage students. A task is worthwhile if it supports some or all of the following:

- multiple strategies can be used to solve the problem.
- important, useful, or “real world” mathematics is embedded.
- different solutions are possible.
- different decisions can be made and defended.
- student engagement and discourse is encouraged.
- conceptual development is promoted.
- grade level skills are required.
- the teacher has an opportunity to assess student performance.

A teacher who exemplifies good problem solving instruction provides opportunities for students to:

Understand the problem. The teacher:
- gives practice in this step alone.
- asks good questions to guide student thinking.
- provides adequate structure; for example, "I need to ...."
- allows students practice in separating important information from unimportant information.
- tries the "Aldo Bianchi Approach": hand out a problem, give time to read, collect. Draw everything you can from the group, then return the problem. Teach them to underline important parts, take notes, etc.
- provides opportunities for partner work and small group work.
- resists the temptation to formulate the problem for the student.

Devise a plan. The teacher:
- models the thinking process.
- focuses on this step alone.
- teaches specific strategies.
- encourages student questions.
- encourages dialogue among students.
- builds in opportunities for students to share problem solving/inquiry strategies.
- has materials/manipulatives available to students.
- builds in opportunities for students to evaluate their process.
- provides adequate structure to guide students as they learn the process.

Carry out the plan. The teacher:
- asks good questions to guide student thinking.
- honors different approaches.
- teaches students to document their process.
- encourages connections and generalizations.
- encourages the appropriate use of math language and representation.
- teaches students to build a direct link between the solution, representation, and connections.

Look back at the problem. The teacher:
- provides time for looking back.
- encourages students to reread the problem and their response.
- provides time for revision.
- has students assess their own work using a standards-based scoring guide.
- has students assess the work of others using a standards-based scoring guide.
- gives feedback that indicates where improvements could be made on future problems.

The Vermont Framework of Standards and Learning Opportunities is based on the premise that all students can achieve the standards in the Vital Results and Fields of Knowledge. As we work to make this goal a reality, it may help to focus on the characteristics of a student who meets the standard in problem solving.

A student who exemplifies good problem solving skills:

is able to understand the problem and devise a plan. The student:
- reads for understanding
- separates important information from unimportant information
- supplies any missing information
- states the question they have to answer (writes an "I need to..."statement)
- knows problem-solving strategies
- has had many concrete experiences upon which understanding can be built
- has a solid understanding of math concepts
- is skilled in formulating a workable approach

- makes good decisions regarding materials and strategies
- is flexible enough to choose another strategy if needed
- is able to think about a situation without panicking and going straight to an answer

is able to carry out the plan. The student:
- effectively implements an approach
- uses appropriate vocabulary and representation skills
- documents work in a readable manner

is able to reach conclusions. The student:
- perseveres to arrive at a conclusion
- makes observations, connections or generalizations while problem-solving
- rereads the problem to make sure the question(s) was answered

It is evident that students need strong mathematical skills and concepts in order to be successful in the problem solving arena. Administrators and school boards can promote problem solving in many ways. Use the list below to begin a dialogue on improving your school's efforts for its students.

To promote problem solving, administrators:

support teachers by:
- providing quality professional development in mathematics
- encouraging teachers to take courses and to attend local, state and national mathematics conferences, workshops and network meetings
- adopting and implementing a standards-based mathematics program
- supporting teachers as they implement the program
- developing an articulated local curriculum and assessments that support the standards and take into account the grade level expectations of the New Standards Reference Exam

work with the public by:
- promoting the use of the New Standards released tasks and practice tests to give students experience with the testing situation
- using assessment data to create an action plan to improve student performance
- holding family/community math events to highlight local curriculum, student work and progress
- discussing characteristics of problem solving and the Vermont Portfolio Scoring Guide

ASSESSING PROBLEM SOLVING

Assessment that enhances the learning of mathematics is a regular part of classroom activity and does not simply mark the end of a learning cycle. Quality assessments are learning opportunities in addition to being a forum for students to show what they know and can do. Students will be learning mathematics as they are assessed, and will grow in their independence as learners, particularly when self assessment using the Vermont Mathematics Scoring Guide has been a routine part of their problem solving work.

Assessment of student work in problem solving is important to continual student improvement. That assessment must be intentional and on-going, and should include informal teacher feedback, student self assessment, and formal assessment. To effectively influence student confidence and ability to solve problems in various contexts, teachers need to provide students with consistent and timely feedback about the results of their problem solving. Assessment feedback is given using the Scoring Guide, and in written and oral comments by the teacher. The Scoring Guide is designed to reflect a student's ability to perform on all aspects of problem solving.[1]

Formal assessment of student problem solving abilities provides an opportunity for students to show what they know and can do without the aid of group instruction or teacher assistance. Caution must be exercised by teachers to preserve the independence of student thinking in formulating the problem, carrying out their approach and reaching a solution. It may be necessary to place limitations on the assessment environment to ensure independent student work. Teacher feedback should be limited to genuine questions which do not lead students to strategies or solutions.[2] Occasionally students may complete a formal assessment with a partner, but most formal assessment work will be done independently.

On-going informal assessment, student self assessment and more formal assessments are all necessary to provide students the opportunity to become independent learners and to build their mathematical power. Using this combination of assessment formats, teachers and students will easily see growth in student skill, conceptual understanding and problem solving abilities.

1 (1) See "Explanation of VT Scoring Guide," available from the VT Dept. of Education.

2 (2) See Appendix B for generic questions.

Comparing the Vermont Mathematics Portfolio and the New Standards Reference Exam

A closer look at assessing problem solving as it appears in the Vermont Mathematics Portfolio and the New Standards Reference Exam reveals that both share the same content and intent, but the process differs. The chart below may help clarify the similarities and difference

Vermont Mathematics Portfolio	New Standards Reference Exam
• The Vermont Mathematics Portfolio assesses approach and reasoning, connections, solution and communication through language, representation and documentation.	• The New Standards Reference Exam assesses approach and reasoning and solution.
• The teacher may read, clarify, or paraphrase the problem to enable the student to understand the given problem.	• Student reads to understand the given problem.
• Many problems lead to the formulation of a general rule or other connection.	• In 25% of the problems, students are expected to formulate a general rule.
• The level of skills and conceptual understanding needed to solve the problem may be moderate to high.	• The level of skills and conceptual understanding needed to solve the problem is low to moderate.
• The Vermont Portfolio is a collection of student work over time: as much as two years.	• Problems are solved 'on demand', giving a snapshot of student performance.
• A portfolio problem could be completed in as little as one class period, or as long as one week or more; some teachers assign problems as out-of-class work.	• Students are given at least 15 to 45 minutes to solve a problem.
• Some portfolio problems can be completed with a group or a partner.	• Problem solving is always individual work.

Elementary Level

Understanding the differences between tasks designed to assess skills, concepts and problem solving.

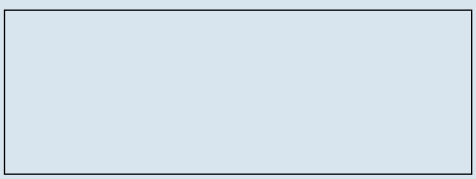
The following examples illustrate the differences between tasks that assess skills, concepts or problem solving. Skills and concepts tasks **are not** appropriate portfolio problems and **should not** be used in students' portfolios.

Skills

Elementary Level

Finding the perimeter is an example of a **skills** task that falls under VT standard 7.7f, *students measure as accurately as possible or round off, as appropriate,* and also includes the New Standards Performance Standard *refers to geometric shapes and terms correctly with concrete objects or drawings*.

What is the perimeter of the rectangle in inches and centimeters?



Perimeter = __________ inches

Perimeter = __________ centimeters

This task is designed to assess if students know the meaning of the term perimeter, to find out if they can measure the perimeter in both customary and metric systems, and round to the nearest whole unit.

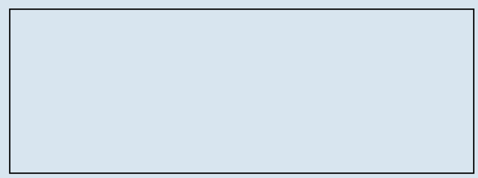
Concepts

Elementary Level

Fence Me In focuses on the **conceptual understanding** of area and perimeter, which falls under Vermont Standard 7.7d, *students begin to use simple concepts of scale, using combinations of units and the relationships between area, perimeter, and volume.*

Fence Me In

The drawing below shows a fenced-in garden plot that is 3 meters wide and 8 meters long. Find its area and perimeter.



Area = ______________________________

Perimeter = __________________________

In the space below, design a second garden plot that uses less fencing, but provides more area. Be sure to label the length and width.

Now design a third garden plot that uses more fencing, but provides less area. Be sure to label the length and width.

This task gives the student an opportunity to apply the concept of the relationship between area and perimeter. The task requires that the student find the area and perimeter of the given garden plot. Students must then recognize perimeter in the real world context of fencing and change this number to produce an increase and decrease in the area of the garden. This task probes the student's understanding of perimeter and area and can be easily accomplished only if the student understands how area and perimeter relate. The strategy needed to accomplish the task and the skills needed to answer the questions are in the medium to low range of difficulty.

PROBLEM SOLVING

Elementary Level

In the **problem-solving** task, **Pork Chop's Pen**, the student is given the opportunity to choose and use problem solving strategies. The task measures what the student can do with the mathematics that he or she has already learned and falls under the VT standard 7.10a, b, c, d, e. *The student can solve problems by reasoning mathematically with concepts and skills expected in those grades; and the student determines what the question, assignment or problem is really asking them to do; and the student will create and use a variety of strategies and approaches to solve problems, and learn approaches that other people use; and the student will make connections between concepts in order to solve problems; and extend concepts and generalize results to other situations.*

Pork Chop's Pen

Your new puppy, Pork Chop, must be kept in the backyard while you are at school. The puppy loves to play outdoors, so your parents decided to build a pen to allow Pork Chop to be outside while you are at school. They just happen to have 50 feet of fencing in the basement that can be used for the pen.

What are the dimensions of the rectangular play pen with the most space available for the puppy to play?

Write a letter to your parents explaining your choices and which pen you would recommend they build. Be sure to show how you made your decisions and include a mathematical representation to support your solution.

Source: adapted from *Exemplars, A Teacher's Solution, 1996*

To accomplish this task, the student must **formulate** the problem, decide what the problem is asking him or her to do, extract pertinent information from the problem, figure out what additional information is needed and decide on an approach. In this task the student needs to find the largest square or rectangle they can make that has a perimeter of 50 feet. They need to realize that the size of the pen is the area of the pen.

The **implementation**, or approach used to accomplish the task is left up to the student. He or she will bring some math concepts and skills to the problem. The student *might* build pens using string, square tiles or linking cubes, or he or she *might* make drawings to represent the fence and pen. The student *might* choose to organize this information in a table.

Pork Chop's Pen Possibilities

Length (feet)	Width (feet)	Perimeter (feet)	Area (square feet)
24	1	50	24
23	2	50	46
22	3	50	66
21	4	50	84
20	5	50	100
19	6	50	114
18	7	50	126
17	8	50	136
16	9	50	144
15	10	50	150
14	11	50	154
13	12	50	156

The student *might* also choose to increase the area of the pen by using one side of the house as a border.

Finally the student provides closure to the problem solving process by writing a summary statement and coming to a general **conclusion**. No matter what approach is taken, the successful student will recognize that the best way to arrange the fence is as close to a square as possible, because it maximizes the area. Some exceptional students *may* choose to make the sides fractions of feet to get even closer to a square. Still others *may* recognize that if the fencing is flexible the largest possible area would be a circular play area.

It should be evident that in solving Pork Chop's Pen the student has to have both mathematical skill and conceptual understanding to accomplish the task. In addition the ability to formulate the problem, implement an approach and arrive at a conclusion is necessary.

MIDDLE SCHOOL LEVEL

Understanding the differences between tasks designed to assess skills, concepts and problem solving.

The following examples illustrate the differences between tasks that assess skills, concepts and problem solving. Skills and concepts tasks **are not** appropriate portfolio problems and **should not** be used in students' portfolios.

SKILLS
Middle School Level

Solve for x is an example of a **skills** task that falls under VT standard 7.8cc, *students explore solutions of unknown quantities in equations.*

> *Solve the following equation for x:*
>
> $305 = 30 + 15x$

This task is designed to assess if students can use a symbolic method for finding the value of an unknown in an equation. This skill is routine and well practiced in most 6th, 7th and 8th grade classrooms.

CONCEPTS
Middle School Level

The Walkathon focuses on the **conceptual understanding** of linear functions, which falls under VT standard 7.8bb, *students understand variables in simple functions, especially linear and exponential functions, represent relationships with tables, graphs, and verbal or symbolic rules, analyze tables, graphs and rules to determine relationships.*

> **The Walkathon**
>
> *Students participating in a walkathon to benefit the local food shelf, plan to ask their sponsors for a donation of $5.00 and $0.50 for every mile they walk.*
>
> *Write an equation for figuring out the donation, based on the number of miles walked.*

This task gives the student an opportunity to apply the concept of linear equations to a new situation, set in a real world context. The task requires that the student represent the relationship between the number of miles walked and the donation received from the sponsor, in a symbolic form. To accomplish this the student must understand more than how to follow a routine to solve an equation. Then they need to understand how the donation is affected by the variable, distance walked, and how one variable is dependent on another.

PROBLEM SOLVING
Middle School Level

In the **problem solving** task, **Choosing the Best Plan**, the student is given the opportunity to choose and use problem solving strategies. The task measures what the student can do with the mathematics that he or she has already learned and falls under the VT standard 7.10bb, dd and ee. *The student can formulate and solve a variety of meaningful problems, integrate concepts and techniques from different areas of mathematics, and generalize solutions and strategies to new problem solving situations.*

> **Choosing the Best Plan**
>
> *A new movie theater has opened in your town. As a promotion they are offering a yearly membership which costs $50.00 and allows the member to pay just $1.00 for each movie they see. Without a membership, the cost of a movie is $6.00. Your friend Bill, is interested in the membership, but needs your help in deciding whether it is a good deal. He can't remember how many movies he watched last year, but he thinks he saw more than five.*
>
> *Help Bill decide whether he should become a member or not and show your answer clearly.*
>
> Adapted from Moving Straight Ahead, Connected Mathematics

To accomplish this task, the student must **formulate** the problem, decide what the problem is asking him or her to do, extract pertinent information from the problem, figure out what additional information is needed and decide on an approach. In this task the student needs to discover that Bill will need to look at comparative costs for a varying number of movies since he can't remember how many he saw last year. The pertinent information is the cost of the membership and the per movie costs. At some point the student will need to tell Bill to estimate the number of movies he thinks he will see this year.

The **implementation** or approach used to solve the problem is left completely up to the student. He or she will bring some math concepts and skills to the problem. If the student decides on a numerical approach, he or she might choose a number of movies and find the cost under both plans. The student might choose to organize this information in a table.

Choosing the Best Plan

# of Movies	Yearly Membership	Pay as You Go
5	55	30
6	56	36
7	57	42
8	58	48
9	59	54
10	60	60
11	61	66
12	62	72
13	63	78
14	64	84
15	65	90

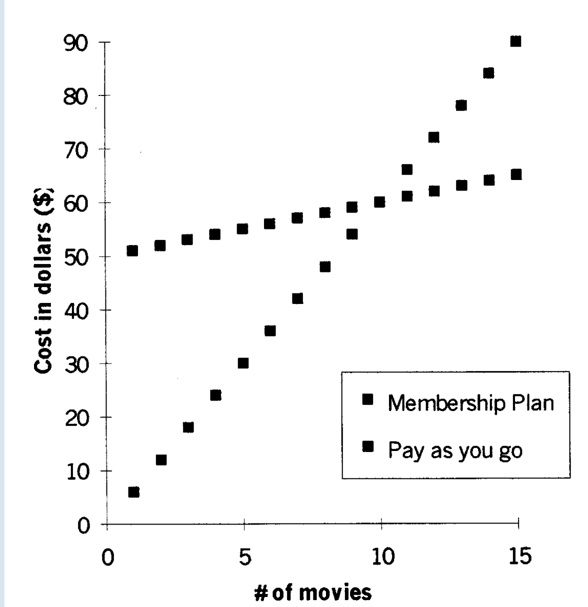
The student *might* bring the concepts of linear functions to the problem and *might* write equations to represent each plan.

Membership plan
C = \$50.00 + x
C= cost
x = # of movies watched

Pay as you go
C = \$6.00x

These plans could be graphed and used to find the "break-even" point.

Choosing the Best Plan



The student *might* also set the two equations equal to one another, understanding that is the point when Bill would spend the same amount of money no matter which plan he chose.

$$C = 50 + x$$
$$C = 6x$$

$$50 + x = 6x$$
$$50 = 5x$$
$$x = 10$$

Finally the student provides closure to the problem solving process by writing a summary statement and coming to a general **conclusion**. No matter what approach is taken, the successful student will recognize that there are times when each plan is cheaper. The student must tell Bill that if he watches fewer than 10 movies, he should pay as he goes. However if he watches more than 10 movies in a year the membership plan is a good deal. In fact, the more movies that Bill goes to the more he will save under the membership plan.

It should be evident that in solving the Choosing the Best Plan task the student has to have both mathematical skill and conceptual understanding to accomplish the task. In addition the ability to formulate the problem, implement an approach and arrive at a conclusion is necessary.

HIGH SCHOOL LEVEL

Understanding the differences between tasks designed to assess skills, concepts and problem solving.

The following examples illustrate the differences between tasks that assess skills, concepts and problem solving. Skills and concepts tasks **are not** appropriate portfolio problems and **should not** be used in students' portfolios.

SKILLS

High School Level

Fair Dice is an example of a **skills** task that falls under Vermont Standard 7.9ddd: *the student uses theoretical probability models to arrive at probabilities for chance events.*

Fair Dice

Two six-sided dice have been altered so that on one of the dice the 4 was replaced by a 3 and the other die has the 3 replaced by a 4. When these dice are rolled, what is the probability that the sum is an odd number?

This task is designed to assess if students know how to find all the possible outcomes of rolling two dice, find the number of outcomes that produce an odd sum and how to express this as a probability using either fractional, decimal or percent notation. This should be a well-practiced routine for high school students. In answering the question the student will either make a sample space or shorten the procedure by realizing that an odd sum is produced when an even number and an odd number are added. The first die has 4 odd numbers and 2 even numbers. The second die has 4 even numbers and 2 odd numbers. This results in a total of 20 possible odd sums out of the 36 possible outcomes.

outcomes = 6 numbers x 6 numbers = 36
odd outcomes = 4 odd x 4 even +
2 even x 2 odd = 16 + 4 = 20
P(odd) = 20/36 = 5/9

CONCEPTS

High School Level Level

What Can Chris Expect? focuses on the **conceptual understanding** of expected value and probability, which falls under Vermont Standard 7.9 dd and ddd: *students use experimental measures of likelihood based on gathering of data to arrive at relative frequencies of chance events, use theoretical probability models to arrive at probabilites for chance events and make predictions based on theoretical possibilities.*

What Can Chris Expect?

Chris beat Casey 70% of the time in arm wrestling over the last three months. The two athletes are having a rematch and Chris will bet $8.00 and Casey bet $3.00 on each game. How much would you expect Chris to win or lose in 50 games? What is Chris' expected value for each game?

This task gives the student an opportunity to apply the concept of expected value in a probability problem. The task requires that the student recognize that the two wrestlers have an unequal chance of winning, but have also wagered unequal amounts. The students must understand that if Chris is expected to win 70% of the time, then Chris should win in 35 out of 50 matches and Casey should win 15 out of 50 matches. The student then needs to apply the amounts bet. Since Chris is expected to win 35 times, then Chris is expected to collect 35 times $3.00, Casey's wager, for a total of $105.00 in winnings. Since Casey can be expected to win 15 times in 50 matches, then Casey should get 15 times $8.00, Chris's wager, for a total of $120.00. Finally the students must state that even though Chris is a better arm wrestler, Chris will be expected to lose $15.00 in 50 matches. The expected value can be found by dividing the expected amount earned by 50 to get the expected value for each match. Chris can expect to lose $0.30 cents per match.

Problem Solving

High School Level

In the **problem solving** task **Jump Ball Probability** the student is given the opportunity to choose and use problem solving strategies. The task measures what the student can do with the mathematics that he or she has already learned and falls under the Vermont Standard 7.10aa, bbb, ccc, eee: *the student can formulate and solve problems in many kinds of situations using grade-related mathematical concepts and reasoning strategies; and formulate and carry out detailed solutions to complex problems, using appropriate problem solving techniques; and carry out a systematic analysis of different possibilities in a complex situation; and work to extend specific results and generalize from them.* The task also falls under Vermont Standard 7.9 ddd: *the student uses theoretical probability models to arrive at probabilities for chance events.*

Jump Ball Probability

One way to handle a "jump ball" situation when we play basketball without a referee would be to determine possession of the ball as follows:

The two players face each other with a clenched fist and on the count of two each extends one, two, or three fingers.

One player is selected to call "odd" or "even" as they both extend their fingers.

The total number of extended fingers are counted and if the count matches the odd or even call, the caller gets the ball. Otherwise, the other team gets it.

The caller will alternate between teams each time there is a new "jump ball" situation.

You need to answer two questions:

1. *Is the odd/even method fair?*
2. *Is there any way that you can improve the odds in your favor when you are involved in a "jump ball"?*

Contributed by Ray Henderson, Missiquoi Valley Union High School, Swanton, Vermont

To accomplish this task, the student must **formulate** the problem, decide what the problem is asking him or her to do, extract pertinent information from the problem, figure out what additional information is needed and decide on an approach. In this task the student needs to find out if the game is fair basing this on the assumption that a game is fair if there is an equal probability to win or lose. The student must also see if a he or she can discover a strategy that will make it more likely to win the game.

The **implementation**, or approach used to accomplish the task is left up to the student. He or she will bring some math concepts and skills to the problem. The student *might* decide to make a sample space to list all the possible outcomes of calling and finger throwing. The student *might* choose to organize this information in a table.

Other Player	**Caller**					
	Even			Odd		
# fingers	1 finger	2 fingers	3 fingers	1 finger	2 fingers	3 fingers
1	Caller	Other	Caller	Other	Caller	Other
2	Other	Caller	Other	Caller	Other	Caller
3	Caller	Other	Caller	Other	Caller	Other

The student would realize that the chances of winning and losing are equal and the game is fair.

The student *might* also solve this problem by thinking about what happens when it is not their turn to call. The outcome depends on what the opponent calls and the finger count. The student may realize that there are six possible outcomes, given the probability of the call is 1/2 either way and the probability that any given number of fingers put out is 1/3.

If the non-calling player extends one finger, and the calling player extends 1 finger and says even, then the caller wins.

The student *might* list all the possible outcomes paired with one finger thrown by the non-caller. (E = even, O = odd, 1,2,3 represent fingers thrown by caller)

P(E-1) = (1/2)(1/3) = 1/6 caller wins
P(E-2) = (1/2)(1/3) = 1/6 caller losers
P(E-3) = (1/2)(1/3) = 1/6 caller wins

P(O-1) = (1/2)(1/3) = 1/6 caller loses
P(O-2) = (1/2)(1/3) = 1/6 caller wins
P(O-3) = (1/2)(1/3) = 1/6 caller loses

The student realizes that both the caller and non-caller win 3 out of 6 times, making the game fair. The student also realizes that the same logic could be used if the non-caller threw 2 or 3 fingers.

The second part of this problem requires the student to see if he or she can improve the chances of winning. The student *might* use the charts or lists created in the first part of the problem to see if they get to call even or odd, they can improve their chances of winning. They *might* notice that if they put out 1 or 3 fingers, the probability that the event will be even is 2/3. If they put out 2 fingers, the probability that the event will be odd is 2/3. The successful student will realize that when they are the caller, if they extend 2 fingers, they should call "odd," and when they extend one or three fingers, they should call "even."

Finally the student provides closure to the problem solving process by writing a summary statement and coming to a general **conclusion**. No matter what approach is taken, the successful student will recognize that if no player sees a "trick" the game is fair. However if one person knows the "trick" then he or she can improve their chances when he or she is the caller to 2/3. The student *might* find the theoretical chance of winning in this circumstance. Since each person is the caller 1/2 the time then:

When the opponent makes the calls the probability that the "tricky" player wins is (1/2) (1/2) = (1/4) and

When the "tricky" player makes the call the probability that he or she wins is (1/2) (2/3) = (1/3)

Since the two events are mutually exclusive we add the probabilities

(1/4) + (1/3) = 7/12

The successful student *might* also confirm their answer by conducting an experiment under both circumstances, neither player knows any tricks, and where one player know the trick and the other does not. The student *might* also try to decide what happens if both players know the trick. The student would compare the experimental results with the theoretical results.

It should be evident that in solving Jump Ball Probability the student has to have both mathematical skill and conceptual understanding to accomplish the task. In addition the ability to formulate the problem, implement an approach and arrive at a conclusion is necessary.

PROFILE OF A COMPREHENSIVE MATHEMATICS PROGRAM

In order for students to be able to achieve the standard on the New Standards Mathematics Reference Exam, they need to have experienced a comprehensive mathematics program that develops and strengthens skills, concepts and problem solving abilities. All are required in the exam and all play a role in the problem solving score.

A comprehensive mathematics program is multifaceted and complex. There are many ingredients which we know must be present and working well in order for our students to perform well in mathematics at the level which we have set for them. This Profile is meant to paint a picture of what that comprehensive program looks like at the classroom level and beyond. It is meant to give a vision of what we are striving to attain in mathematics classrooms, a vision grounded in practical details that we can learn from and begin to apply. This Profile cannot give all the details for each aspect of the program, nor is it meant to; it will focus on problem solving. Building a comprehensive program happens over time, and requires placing priorities on professional development for teachers, guidance from knowledgeable experts, collaboration among staff, attention to assessment data and actively supportive administrators and community.

The elements of a comprehensive mathematics program include :

- Professional development
- The Vermont Framework of Standards and Learning Opportunities
- Classroom Culture
- Instruction
- Tasks
- Assessment

The following profile describes the characteristics of these elements of a comprehensive mathematics program and their role in improving problem solving by providing students the experiences and learning opportunities necessary to meet and surpass the standards:

Professional Development

Professional development in mathematics is a district priority. Teachers are familiar with the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*. They are committed to a mathematics program which reflects these standards and are actively involved in evaluating, adapting and aligning their instruction to the standards.

All teachers are receiving on-going professional development in the portfolio process and use of the Vermont Scoring Guide. Integrating portfolio work into the daily mathematics of the school at all grade levels is providing students opportunities to demonstrate both conceptual understanding and application of the mathematics.

Teachers are supported in continuing their own learning to become a more effective math teacher by enrollment in math courses and by attending local, state, and national mathematics conferences, workshops and network meetings. With the support of their administrators, they are seeking out mentors, acting as mentors and networking with other teachers.

The VT Framework of Standards and Learning Opportunities

An articulated curriculum is in place which is fully aligned with both the VT Framework and the NCTM Standards. Standards-based mathematics programs which align well with the Vermont Framework of Standards and Learning Opportunities have been reviewed and one has been adopted or created. Standards-based assessments are in place which have been aligned with the program, the Framework and the New Standards Reference Exam.

Teachers are committed to the chosen standards-based program and are uniformly implementing it within and across the grades. Teachers have been provided an adequate amount of orientation to the program. A long term professional development plan has been formulated and is being implemented to help teachers make the transition to standards-based instruction . No teacher is expected to “figure it out” alone. Content instruction needs of the staff are being addressed through the professional development plan. Instruction and practice in the pedagogy of inquiry mathematics and constructivism is an integral part of content instruction and professional development.

The Learning Opportunities section of the Vermont Framework guides teachers in implementation of best practices. Students receive instruction in mathematics which is consistent between teachers and is within the continuum set forth in the chosen program. Skill development specified in the local curriculum takes into account the grade level expectations of the New Standards Reference Exam. Local assessments are in place that monitor that skill development.

Classroom Culture

Care is taken to create a rich environment in the mathematics classroom. Classroom practices reflect the changing role of the teacher, facilitating a community of learners actively working to make sense of mathematics. Students discuss, question and challenge ideas. Work is often collaborative, with student helping student. Direct teaching is used when appropriate, with the major emphasis placed on engaging students in learning mathematics. This is done through solving and discussing interesting and relevant problems. The focus of discussions and the tone of the classroom is aimed at understanding mathematics, not simply "doing" mathematics.

Complex problems and thought provoking questions are the norm. Students are expected to solve problems in a variety of ways, and to communicate their mathematical thinking and their solutions in written as well as oral forms. Students are praised for asking questions and for innovative solutions. Risk-taking is valued. There is an expectation that students will learn from the thinking of others as strategies are shared. Students are continually developing their abilities to formulate and implement strategies, to reach conclusions and to communicate solutions to others.

Instruction in Problem Solving

In order to become successful problem solvers, students are given instruction and practice in the problem solving process. For example, students are instructed in a particular strategy, given a variety of problems and asked to practice that strategy. Students are also given a single problem and asked to solve it using a variety of strategies. Through a balance of direct instruction and constructivist learning, students internalize the problem solving process and build a repertoire of effective strategies with which they are competent. Students learn to recognize the mathematical elements of problem situations and match them with appropriate and efficient strategies. They become adept at recognizing when a strategy is not working and they try others.

Instruction in the Vermont Mathematics Scoring Guide occurs in all grades. Students keep portfolios demonstrating their mathematical knowledge in solving problems from all four concept areas: Arithmetic, Number, and Operation, Geometry and Measurement, Function and Algebra, and Statistics and Probability. Students become fluent with the Mathematics Scoring Guide through self-scoring and peer-scoring. Continual progress is made in each of the criteria by providing regular feedback. Students are taught to make connections such as identifying patterns that can be used in similar situations, explaining how the mathematics used in a problem fits their conceptual understanding, or discovering a general rule that applies to other circumstances. Students are regularly exposed to challenging and non-routine problems which may be solved over time.

Students are frequently given additional opportunities to reflect on their mathematical understanding. Thoughtful written reflection is used by students to document their understanding of concepts as they learn them. Students also reflect on skills, procedures, problem solving and attitudes, keeping these reflections to review and update periodically. Student reflections are used by teachers to assess understanding and inform teaching.

Problem Solving Tasks

The mathematics program includes both portfolio tasks and on-demand problem solving tasks which require students to apply well assimilated facts, skills and concepts. On-demand tasks are used to assess individual abilities and are therefore completed alone in a single sitting. They include tasks with medium to low conceptual and skill requirements, but with a significant formulation hurdle. Problem formulation is critical to both the problem solving process and the problem solving score on the New Standards Reference Exam. Therefore, throughout the year students are given tasks in which the most challenging piece for the student is to figure out what is being asked and how to formulate an approach, before deciding on the needed skills and concepts.

These on-demand tasks contrast with the portfolio-type tasks which may be completed over time, may be completed by a group or with input from other students, may include revisions, and may include more challenging facts, skills, concepts and language than the on-demand tasks.

The Importance of Assessment

Teachers vary from the usual pattern of teaching a topic and assessing it, then teaching another topic and assessing that. Instead, teacher assessment includes problems which require students to use the math learned 3, 6, 12 or even 24 months previously. As a result, students receive practice selecting and deploying skills and tools that are not part of the current routine but which are needed to solve the problems. Students are encouraged to formulate an approach based on all the math in their repertoire, and receive practice doing so.

Students are actively involved in designing problems. They increase their conceptual understanding by creating questions or tasks that assess targeted mathematical concepts, and create sample solutions for those tasks. By constructing questions, students are encouraged to think more deeply about the concepts they are learning and their sample solutions make sure the questions are actually doable. Throughout the year students use the New Standards release tasks and practice tests, scoring their own or their peer's responses compared to anchor papers. These experiences deepen their understanding of quality work and the standard they are striving to achieve.

Attention is paid to the balance between instruction and standards-based assessment, with the Vermont Framework, the local curriculum and the chosen program defining the desired outcomes. A variety of assessments are used, including teacher-developed, program-provided, local and state assessments. Information from each of these assessments is regularly analyzed and used in decision making that influences teacher judgment about program and classroom instruction. The process continues, spiraling back to improve teacher effectiveness and student performance.

SUMMARY

The profile above describes many of the ingredients present and working well in a comprehensive mathematics program. Commitment to this profile can guide ongoing progress toward fulfillment of this vision in your school. Your community can have confidence and take pride in knowing that students are experiencing a rigorous mathematics program which is a bridge to real world problem solving, job skills and responsibilities. Building this kind of a program happens over time, and with the investment of all key stakeholders within a school and community.

In addition to the resources within your school and community, many other resources exist to help you implement a comprehensive mathematics program in your school, including VISMT, the State Department of Education, and private consultants. See the contact list below for helpful names and phone numbers.

VISMT

Jim Abrams, *Director of Mathematics*
828-0069

Nicole Saginor, *Teacher Associate Director*
828-0068

VT Dept. of Education

Deb Armitage, *Assessment Consultant*
828-5409

Aldo Bianchi, *Assessment Consultant*
828-5408

APPENDIX

Appendix A:

Glossary of Terms

Articulated curriculum - a curriculum which provides a clear description of the content and skills, instructional guidelines, and the assessment tools and plan that provide students the opportunity to learn and perform in relation to the standards.

Conceptual understanding - the understanding of the idea underlying a concept. Tasks which assess conceptual understanding often require students to analyze an idea, reformulate it and express it in their own terms.

Connections - mathematics is an integrated field of study and not a list of strands or standards. Students need instruction and opportunities to recognize and use connections among different mathematical ideas; to understand how these ideas build upon one another to make a coherent whole; and to recognize, use, and learn about mathematics in contexts outside of mathematics.

Constructivist learning – "The basic tenet of constructivism is simply this: Children construct their own knowledge. Construction requires tools. The tools children use to construct knowledge are the ideas they already have. To use ideas to construct new ideas means that children must be mentally engaged in the act of learning. They must call up those ideas that are relevant and use them to give meaning to the new or emerging or changing ideas that they are developing.

Since ideas are used to develop new knowledge, those existing ideas will necessarily be connected to the new idea. The result is a network of meaningful, related, useful ideas. The more ideas and the more interconnected they are, the better all of the ideas are understood. Things make more sense when they can be related to a lot of the ideas already understood." (Van de Walle, 1999)1

Generalization - creation of a general rule that can be used no matter what the numbers in the problem are, either expressed in algebraic terms or words.

On-demand tasks - a type of performance assessment that takes place at a point in time and over a limited amount of time. State tests, SAT's, and most final exams are types of on-demand assessment.

Polya - George Polya (1887-1985) was a Hungarian mathematician known for his work in probability, analysis, number theory, geometry, combinatorics and mathematical physics. He wrote a book, *How to Solve It,* which outlined the process an individual uses when solving a non-routine problem. Polya gave wise advice: "If you can't solve a problem, then there is an easier problem you can't solve: find it."

Problem conclusion - providing closure to the solution process through summary statements and generalizations, looking back at the problem.

Problem formulation - understanding the problem, deciding what question needs to be answered, separating important and unimportant information and devising a plan for solving the problem.

Problem implementation - making basic choices involved in planning and carrying out a solution, including using and inventing a variety of approaches, invoking problem solving strategies, breaking a problem into simpler parts, and integrating concepts and techniques from different areas of mathematics.

Significant formulation hurdle - refers to a problem in which the most difficult task required of a student is to understand the problem and devise a plan. Tasks which are used to assess problem solving have significant formulation hurdles.

Standards-based program - uses a curriculum designed with a specific focus on the standards. Cumulatively, across all learning opportunities and units of study, as well as their related assessments, all students have access to and demonstrate attainment of the knowledge and skills identified in the standards.

Task-specific scoring guides - a set of scoring guidelines that is specific to a particular task. The criteria are addressed and described in terms of specific content or capacities that can be demonstrated in terms of particular, identified content relevant to the task.

1 (1) Van de Walle, John A., 1999, "Reform Mathematics vs. The Basics: Understanding the Conflict and Dealing with It"

Appendix B:

GENERIC QUESTIONS

Problem Comprehension:

What is the problem about? What can you tell me about it?

How would you interpret that?

Would you please explain that in your own words?

What do you know about this part?

Do you need to define or set limits for the problem?

Is there something that can be eliminated or that is missing?

What assumptions do you have to make?

Approach and Reasoning:

Where could you find the needed information?

What have you tried so far? What steps did you take?

What didn't work? How did you decide that? What did you discover?

What do you understand now that you didn't before?

How did you organize the information? Do you have a record?

Do you have a system, a strategy?

How would you research that?

Have you solved any problems like this before?

Give me another related problem. Is there an easier problem?

Is there another way to solve this problem?

Is there another way to communicate that?

What were your first thoughts about the task?

Are there any relationships in this problem that will always be present in similar problems?

Are there other ways to solve this problem?

Connections:

Can you predict what will happen?

What was your estimate or prediction? Why?

How do you feel about your answer?

What do you think comes next?

What else would you like to know?

Is there a general rule?

Is the solution reasonable?

What made you think that was what you should do?

How is this like the mathematics of a real-life problem?

What were the mathematical ideas in this problem?

What was the one thing you learned?

What kinds of mathematics were used in this investigation?

Where else would this strategy be useful?

What other problems does this lead to?

How would this work with other problems?

What questions does this task raise for you?

Solution:

Are you sure your solution is correct? Why/How?

Is that the only possible answer?

How would you check the steps you have taken for your answer?

Other than retracing your steps, how can you determine if your answers are appropriate?

Is there anything you overlooked?

How did you know you were done?

Communication:

Could you explain what you know right now?

How would you explain this process to a younger child?

Could you write an explanation for next year's students (or some other audience) of how to do this?

Which words were most important? Why?

Could you explain that another way?

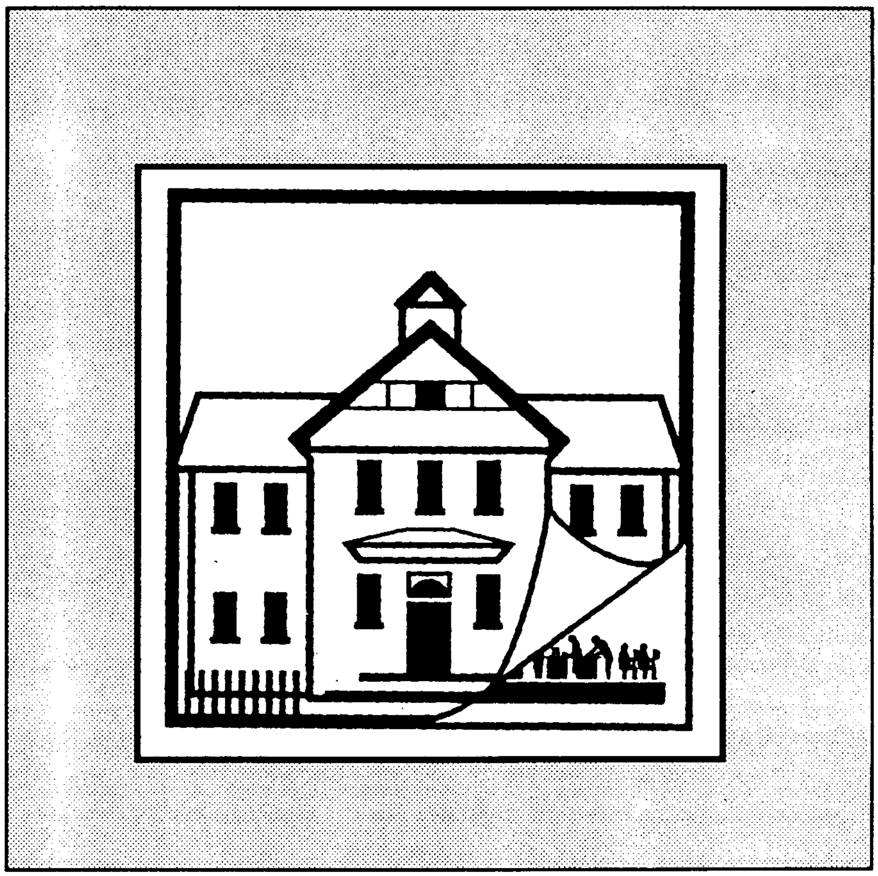
Can you explain your reasoning?

What pictures do you have in your mind to help you think about the task?

Appendix C:

MATHEMATICS SCORING GUIDES

Vermont Elementary and Middle Level Mathematics Portfolio Scoring Guide



This scoring guide was developed by
the Vermont Department of Education,
in collaboration with the
Vermont Institute for Science, Math and Technology.

Problem Solving Criteria

Approach and Reasoning

START HERE →

Level 1	Level 2	Level 3	Level 4
• Approach wouldn't work **or** • No approach evident	• Approach would[1] lead to solving only part of the problem[2] or reaching a partial solution **or** • Approach would work but there is some flaw in the reasoning	• Approach worked or would[1] work for solving the problem, and reasoning, if evident, is not flawed (Note: Use of a formula is an approach that worked or would work)	Approach worked, and at least one of the following 3 additional aspects of good problem solving is evident. • **Justifying the application of a known formula or rule** used to solve all or part of the problem **or** • **Making a formula or rule** used to solve all or part of the problem **or** • **Describing verification** of her/his solution[3]

Connections

← START HERE

Level 1	Level 2	Level 3	Level 4
• Response stopped without including a mathematically relevant observation with respect to her/his solution	• Made a mathematically relevant **observation** about her/his solution **or** • Identified an **underlying** mathematical **concept or pattern** in her/his solution **or** • Solved the problem and then **recreated**[4] the problem and found a new solution **or** • Solved the problem and then used a **different** mathematical **process** to solve the same problem	• Related this problem to a similar problem or to a real world phenomenon **by expressing** the **mathematical relationship(s)** **or** • **Analyzed the relationship** among elements in her/his solution or among similar or different mathematical topics in her/his solution **or** • **Tested** and accepted and/or rejected an **hypothesis or conjecture** about her/his solution **or** • Identified a formula or rule, while solving the problem, that worked or would work in solving all or part of **that** problem.	• Solved the problem, discovered a general rule[5] about the solution[3], and **demonstrated understanding of the generalization** either through explanation of the derivation, or through application to more than one other case **or** • Solved the problem, and then **extended** her/his solution to a more **complicated situation** **or** • **Evaluated** the **reasonableness** or **significance** of her/his solution

Solution

← START HERE

Level 1	Level 2	Level 3
• No work is present **or** • No part of the solution[3] is correct **or** • Some work is present, but the work doesn't support the answer given	• The solution[3] is correct for only part of the problem, and there is work to support those correct part(s) **or** • The solution[3] contains mathematical errors which lead to an incomplete or incorrect answer	• The answer is correct, and the work in the solution[3] supports the answer

1 **Would:** An approach that would work for solving the problem addresses all aspects of the mathematical situation presented in the task. An approach that would work may contain mathematical errors, an incorrect solution, or may be incomplete.

2 **Part of the Problem:** Within a problem, there may be several mathematical components that need to be addressed, or there may be multi-parts. If not all of the mathematical components of the problem are addressed, or not all of the parts of the problem are addressed, then the student only found an approach to solve part of the problem.

3 **Solution:** All of the work that was done to solve the problem, including the answer.

4 **Recreated:** The student substituted different numbers in the same problem and found another solution, or used the same procedure in a different circumstance

5 **General Rule:** A rule that can be used no matter what the numbers in the problem are, either expressed in algebraic notation or in words.

September 1997

Communication Criteria

Mathematical Language: Terms/vocabulary and symbolic notation

START HERE →

Level 1	Level 2	Level 3
• Is absent **or** • Contains significant flaws in accuracy **or** • Is limited to the language of computation vocabulary and notation **or** • Is limited to formulas that appear without explanation, derivation, or use	Is **relevant,** but may contain minor flaws **and** • Is the **sparse use** of the language of... • Number sense and numeration, number relationships, number systems and number theory (including fractions and decimals) **or** • Geometry and measurement[6], **or** • Statistics and probability, **or** • Patterns, functions, and algebra **or** • Demonstrates understanding of non-computational language presented in the task (Note: Use of a single non-computational term rarely merits a level 2)	Is relevant and contains no significant flaws **and demonstrates understanding through...** • **Consistent use** of non-computational language beyond that presented in the task, including the language of... • Number sense and numeration, number relationships, number systems and number theory (including fractions and decimals) **or** • Geometry and measurement[6] **or** • Statistics and probability, **or** • Patterns, function, and algebra **OR** • **Use of** algebraic or other notation(s)[7]

Mathematical Representation: Graphs, plots, charts, tables, models and diagrams

← START HERE

Level 1	Level 2	Level 3
• **Didn't** attempt to make any mathematical representations to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution **or** • Made only **inappropriate** mathematical representations to solve or communicate an aspect of her/his solution regardless of the correctness of the solution	• **Attempted** to make an appropriate mathematical representation to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution, but the representation lacks labels and/or accuracy with regard to the student's solution. (Note: Completion of a teacher structured representation cannot earn above a level 2)	• Made an **appropriate** and **accurate**[8] mathematical representation to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution. See glossary for requirements. (Note: The student's text may supply the necessary labeling).

Documentation

START HERE →

Level 1	Level 2	Level 3
• The documentation of the student's correct or incorrect solution contains **little or no evidence** of how the problem was solved <u>or</u> the reasoning used	• The documentation of the student's correct or incorrect solution **contains some clear parts,** but there are gaps in how the student solved the problem <u>or</u> the reasoning used	• The documentation of the student's correct or incorrect solution **clearly shows how the problem was solved, and the reasoning used.** This may be evident by some of the following... • Results of any necessary computation are present • Answers are highlighted • Presentation is in logical order • Representations are linked to text • All parts are connected and labeled

[6] **Measurements:** Attributes of length, capacity, weight, mass, area, volume, time, temperature, and angle

[7] **Notation:** Includes the use of algebraic equations and formulas (with all variables defined), and/or other notations (!, Σ, and exponential notations)

[8] **Accurate:** Mathematical representations that are technically correct and executed properly, including labels. See over.

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Mathematical Representation Glossary

In order to ensure that student responses receive a "level 3" on the portfolio scoring criteria for mathematical representations, they should reflect the descriptors stated below. Note: On all representations, labeling may serve as a title. The student's text may also supply the necessary labeling.

GRAPH: A diagram showing a relationship between 2 variables.
A graph should be accurate, appropriate to the task, have a title and correctly scaled and labeled axes or sectors, with data accurately recorded.

PLOT: Stem and Leaf Plot, Scatter Plot, Line Plot, and Box and Whisker Plot.
A plot should be accurate, appropriate to the task, have a title, and any necessary keys and/or scales. Note: Data used in creating a plot should be clearly stated and/or listed within the text.

CHART: Information displayed in rows and columns with no particular order.
A chart should be accurate, appropriate to the task, have a title, labeled rows and columns, and any necessary keys.

TABLE: A systematically ordered chart.
A table should be accurate, appropriate to the task, have a title, labeled rows and columns, and any necessary keys.

Special Case:

Systematic List :	A list of information that is organized systematically
Level 3:	Is systematically organized, accurate, appropriate, labeled, and has any necessary keys
Level 2:	Is organized and appropriate, but lacks accuracy, labels, and/or necessary keys
Level 1:	A labeled list lacking systematic organization

DIAGRAM: An explanatory drawing.
A diagram should be appropriate to the task, explanatory in nature, and have a title, labels, and any necessary keys.

MODEL: A representational drawing or construction, such as sets of plans, scale representations or structural designs.
A model should be accurate, appropriate to the task, have a title and any necessary keys.

Vermont High School Mathematics Portfolio Scoring Guide

Problem Solving

PS1: The Approach (and Reasoning)

The strategies and skills used to solve the problem, and reasoning that supports the approach, if evidenced

Level 1	Level 2	Level 3	Level 4
The approach is not apparent or wouldn't work.	The approach would lead to solving only part of the problem or reaching a partial solution, or, the approach would work, but the reasoning that is evident is flawed.	The approach **worked**, or **would work** for the problem, **and** reasoning, if evident, is NOT flawed.	The approach(es) worked, and additional aspects of good problem solving are evident, such as: • use of multiple approaches for verification, or • use of estimation and comparison, or • testing, accepting, and/or rejecting hypothesis, or • derivation of a general rule as an approach to solving the problem, or • identification of (an) additional factor(s) that MAY affect the solution of a problem (The effect of the(se) additional factor(s) on the solution **must** be demonstrated.). • or other

Approach: The strategies and skills used to solve the problem

Would: An approach that **would** work for a problem, even if not accurately executed, can lead to a Level 3.

Part of the Problem: Some tasks are multi-part. To score at a Level 3 , an approach that would work must be present for each part of the problem.

Reasoning may be seen in the logical sequence of the solution, explanations for decisions, evidence of inductive or deductive reasoning, etc.

Rules:
- The **absence** of reasoning is not penalized in PS1, but if reasoning is present, it must not contain flaws to attain a Level 3.
- A response must meet the requirements of Level 3 before it may be a Level 4

PS2: What – Execution of the Task

The mathematical work that supports the student's answer to the problem

Level 1	Level 2	Level 3
No execution is present, or execution is all wrong, or execution is present, but doesn't support the answer given.	Execution is present for only part of the problem, or execution of the problem is present, but is only partially correct, or partially complete (which may lead to an incomplete or incorrect answer).	Execution of the problem is present and supports the answer, and the answer is correct.

Rule: If a student solves a different problem from the original problem, then s/he scores at Level 1 on this criterion, unless the solution is based upon additional factors a student identifies correctly.

PS3: So What

Demonstration of observations, connections, applications, extensions, and generalizations

Level 1	Level 2	Level 3
Response stops without including a mathematically relevant observation, connection, application, generalization, or extension.	Response includes... • a mathematically relevant observation, or • a mathematically relevant connection, application, extension, or generalization, but does not fully demonstrate an understanding.	Response includes and demonstrates a full understanding of a connection, or a real world application; **or**, makes and justifies a generalization, or an extension of the solution to a more complicated situation.

Connections: Connections are between mathematical ideas, to other problems, to other disciplines, or to similar contexts.

Extension: A student takes an idea from the problem and uses the idea to solve a similar but more complicated problem.

Generalization: One type is a student derived formula, written explanation, or equation that can be used for any value X, not just the values used in the problem. Equations that are only applicable to the data given in the problem are **not** generalizations.

Demonstrate a Full Understanding: There must be **evidence in the work** that the student derived a generalization, or that the student has a full understanding of connections, applications, and extensions s/he made to score at a Level 3.

Rules: A response **scores** at a Level 2 if the scorer knows that the connection, application, extension, or generalization is mathematically relevant, but the student does not fully demonstrate an understanding.

Communication

C1: Mathematical Communication *The use of accurate and appropriate mathematical vocabulary and mathematical representation (i.e. tables, graphs, diagrams, modeling, algebraic equations, formulas, notations...) in communicating the solution*	Level 1	Level 2	Level 3	Level 4
	Mathematical communication is absent, or consistently inaccurate or inappropriate; or the ONLY mathematical communication used is basic vocabulary and/or basic notation.	Mathematical communication is appropriate but contains some flaws in accuracy.	Mathematical communication is appropriate and accurate.	Additional aspects of good mathematical communication are evidenced with **rich** mathematical communication, including the accurate and appropriate use of **symbolic or formal notation**, plus <u>one or more</u> of the following: • accurate and appropriate use of more than one distinctly different representation with a clear linkage between the representations and the text, or the representations with each other, • a clear link between an equation or formula, and a model, diagram, or graph, • other
	If ONLY mathematical vocabulary is used to communicate the solution...			
	the vocabulary is basic, or the vocabulary used is that which is given in the task, but the use is inaccurate or inappropriate.	the vocabulary is beyond basic, but only that which is given in the task, and the use is accurate and appropriate.	the vocabulary is beyond basic, and beyond that which is given in the task, and the use is accurate and appropriate.	

Mathematical Representations: include graphs, numerical tables, models, diagrams, algebraic equations, formulas, or other mathematical notations
Formal or Symbolic Notation: includes the use of algebraic equations, formulas and other notations...
Appropriate: makes sense for the problem
Basic: includes arithmetic related vocabulary and notation
Tables: organized in columns and rows and require labels

Accuracy: Accuracy implies
- the correct use of vocabulary and symbolism
- graphs and tables accurately and completely labeled and scaled
- units of measure present and appropriate for the solution

Rules:
- A response must meet the requirements of Level 3 before it may be a Level 4
- A few accuracy flaws/mistakes, in a response rich with mathematical communication, does not lower the score to a Level 2 unless the mistake(s) are significant in the context of the problem.
- A student does NOT receive credit for completing a representation that is included in the task.
- A student does NOT receive credit for completing a specific type of representation that is called for in the task (e.g. make a scatter graph to represent the data).

C2: Overall Presentation *Effective communication of how the problem was solved and the reasoning used*	Level 1	Level 2	Level 3
	The presentation of the solution contains little or no evidence of how the problem was solved or the reasoning used.	The presentation contains gaps in how the problem was solved or in the reasoning used.	The presentation of the solution clearly shows a connection between steps or ideas that show how the problem was solved and the reasoning used.

Rule: The problem does not need to be solved correctly to score a Level 2 or a Level 3 in C2. This criterion assesses how effectively the student communicates how the problem was solved and the reasoning used.

Reasoning (correct or incorrect) can be seen in the following ways: logical sequence in the solution, the presence of explanations for decisions, evidence of deductive or inductive reasoning, etc.